JEE – Advanced 17th May 2026

Paper 01

Question paper and Solution

MATHEMATICS

SECTION 1 (Maximum Marks: 12)

This section contains **FOUR (04)** questions.

- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered); Negative Marks: —1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks; choosing ONLY (A) and (B) will get +2 marks; choosing ONLY (A) and (D) will get +2 marks; choosing ONLY (B) and (D) will get +2 marks; choosing ONLY (A) will get +1 mark; choosing ONLY (B) will get +1 mark; choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get –1 marks.

SECTION 3 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

SECTION 4 (Maximum Marks: 16)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 ONLY if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks: −1 In all other cases.

Section 1

Multiple choice questions with one correct alternative

1. Consider the function $f:(0,\infty)\to(-\infty,\infty)$ given $f(x)=\sqrt{x}\log_e(x)-x+1$

Then which one of the following statements is TRUE?

(A) The derivative of the function f is decreasing in the interval (0, 1)

(B) The function f has a local maximum at some point $a\in(0,\infty)$

(C) The function f has a local minimum at some point $b \in (0, \infty)$

(D) The function f has NEITHER a point of local maximum NOR a point of local minimum in the interval $(0, \infty)$

Ans (D)

$$f'(x)=\frac{\sqrt{x}}{x}+\frac{1}{2\sqrt{x}}\ln x-1=\frac{1}{\sqrt{x}}+\frac{\ln x}{2\sqrt{x}}-1$$

$$f''(x)=-\frac{1}{4}x^{-\frac{3}{2}}\ln x+\frac{1}{2}x^{-\frac{3}{2}}-\frac{1}{2}x^{-\frac{3}{2}}=-\frac{1}{4}x^{-\frac{3}{2}}\ln x$$

$f''>0\ \forall\ x\in(0,1) \Rightarrow f'(x)$ is increasing in the interval (0, 1)

$f'(1)=0=f''(1) \Rightarrow$ it is a point of inflexion.

So, no local minima or maxima.

2. Let P be the point on the parabola $y = x^2$ such that the slope of the tangent to the parabola at the point P is 4. Let Q be the point in the fist quadrant lying on the circle $x^2 + y^2 = 2$ such that the slope of the tangent to the circle at the point Q is –1. Let R be the point in the first quadrant lying on the ellipse $x^2 + 4y^2 = 8$ such that the slope of the tangent to the ellipse at the point R is $-\frac{1}{2}$. Then the radius of the circle passing through the points P, Q and R is

(A) $\sqrt{10}$ (B) $\sqrt{5}$ (C) $\sqrt{\frac{5}{2}}$ (D) $2\sqrt{5}$

Ans (C)

Parabola: $\frac{dy}{dx} = 2x = 4 \Rightarrow x = 2 \Rightarrow P(2, 4)$

Circle: $\frac{dy}{dx} = -\frac{x}{y} = -1 \Rightarrow x = 1 \Rightarrow Q(1, 1)$

Ellipse: $\frac{dy}{dx} = \frac{-x}{4y} = -\frac{1}{2} \Rightarrow R(2, 1)$.

$PQ = \sqrt{10}$, $PR = 3$, $QR = 1$

$\Delta = \frac{1}{2}|2(1-1) + 1(1-4) + 2(4-1)| = \frac{3}{2}$

$R = \frac{abc}{4\Delta} = \sqrt{\frac{5}{2}}$

3. Which one of the following matrices can be obtained by performing elementary row transformations on the 3 × 3 identity matrix?

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$

Ans (B)

Only invertible matrices are obtained by row transformations.

Hence checking determinants,

(A) $|A| = 0$

(B) $|B| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 0 & 1 & 0 \end{vmatrix} = -(4-2) = -2 \neq 0$

(C) $|C| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 4 \\ -3 & -3 & 8 \end{vmatrix} = 0$

(D) $|D| = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -2 & -1 & 2 \\ -2 & -1 & 3 \end{vmatrix} = 0$

4. Considering only the principal values of the inverse trigonometric functions, the value of

$$\cot^{-1}(\cot(-11)) + 10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10\sin\left(2\tan^{-1}(2)\right) \text{ is}$$

(A) $3\pi + 7$ (B) 7 (C) $4\pi + 7$ (D) $3\pi - 5$

Ans (C)

$$\pi - \cot^{-1}(\cot 11) + 10\sin\left(\frac{\pi}{2}\right) + 10\frac{\left(2\tan\left(\tan^{-1}2\right)\right)}{1+\tan^2\left(\tan^{-1}(2)\right)}$$

$$= \pi - \cot^{-1}\left(\cot\left(3\pi + (11-3\pi)\right)\right) + 10 + 10\left(\frac{4}{4+1}\right)$$

$$= \pi - (11-3\pi) + 18 = 4\pi + 7 .$$

Section 2
Multiple choice questions with one or more than correct alternative/s

5. Suppose that Box I contains 6 red balls and 9 green balls and Box II contains 8 red balls and 12 green balls. All the balls of Box I and Box II are mixed together and a ball is choosen at random from them. Let E_1 be the event that the ball chosen belonged to Box I and let E_2 be the event that the ball chosen belonged to Box II. Let F_1 be the event that the ball chosen is red and let F_2 be the event that the ball chosen is green Then which of the following statements is (are) TRUE?

(A) The events E_1 and F_1 are independent

(B) The events E_2 and F_2 are dependent

(C) The conditional probability $P\left(\frac{F_1}{E_1}\right)$ is equal to the conditional probability $P\left(\frac{F_1}{E_2}\right)$

(D) The conditional probability $P\left(\frac{F_1}{E_1}\right)$ is greater than the conditional probability $P\left(\frac{F_2}{E_2}\right)$

Ans (A) and (C)

$$P(E_1) = \frac{15}{35} = \frac{3}{7},\ P(E_2) = \frac{20}{35} = \frac{4}{7}$$

$$P(F_1) = \frac{14}{35} = \frac{2}{5},\ P(F_2) = \frac{21}{35} = \frac{3}{5}$$

(A) $P(E_1 \cap F_1) = P(E_1)\cdot P(F_1) \Rightarrow$ independent

(B) $P(E_2 \cap F_2) = P(E_2)P(F_2) \Rightarrow$ independent

(C) $P(F_1 | E_1) = \dfrac{P(F_1 \cap E_1)}{P(E_1)} = P(F_1)$

$P(F_1 | E_2) = \dfrac{P(F_1 \cap E_2)}{P(E_2)} = P(F_1)$

(D) Since the events are independent, $P(F_1 | E_1) = P(F_1)$ and $P(F_2 | E_2) = P(F_2)$.

6. Let P be the plane such that it contains the straight line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z+2}{1}$ and is perpendicular to the plane x + 2y + 3z = 4. Let P_1 be the plane which passes through the point (4, 2, 2) and is parallel to P. Then which of the following statements is (are) TRUE?

(A) The equation of the plane P is 7x – 5y + z = –10

(B) The distance between the planes P and P_1 is 30

(C) The distance of the plane P from the origin is $2\sqrt{3}$

(D) The acute angle between the plane P and the plane 2x + 2y + z = 3 is $\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

Ans (A) and (D)

Dr's of the normal to P is

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 2 & 3 & 1\end{vmatrix}$$

$\vec{n}=-7\hat{i}+5\hat{j}-\hat{k}\Rightarrow(7,-5,1)$

$P:7(x-1)-5(y-3)+1(z+2)=0$

(A) $7x-5y+z+10=0$

(B) $P_1:7(x-4)-5(y-2)+(z-2)=0$

$7x-5y+z-20=0$

Distance between P_1 and $P=\left|\frac{10+20}{\sqrt{7^2+5^2+1}}\right|=\frac{30}{\sqrt{75}}\Rightarrow 2\sqrt{3}$

(C) Distance of P from origin: $D(0,0)=\left|\frac{10}{7^2+5^2+1}\right|=\frac{10}{5\sqrt{3}}\Rightarrow\frac{2}{\sqrt{3}}$

(D)Angle between $2x+2y+z=3$ and $7x-5y+z+10=0$

$\cos\theta=\left|\frac{14-10+1}{3\cdot 5\sqrt{3}}\right|=\frac{1}{3\sqrt{3}}\quad\therefore\theta=\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

7. Let R denote the set of all real numbers. Let f : R → R be an arbitrary function and let g : R → R be the function defined by g(x) = xf(x), for all x ∈ R.

Then which of the following statements is (are) TRUE?

(A) The function g is always continuous at x = 0

(B) If f is continuous at x = 0, then g is differentiable at x = 0

(C) If g is differentiable at x = 0, then f is continuous at x = 0

(D) If g is differentiable at x = 0, then $\lim_{x\to 0} f(x)$ exists

Ans (A), (B) and (D)

$f:R\to R$

$g:R\to R$

$g(x)=xf(x)\ \forall\ x\in R$

(A) Since f is continuous, g(x) = xf(x) is always continuous

$\Rightarrow\lim_{x\to 0^-}g(x)=\lim_{x\to 0^+}g(x)=g(0)=0$

(B) f is continuous at x = 0 $\Rightarrow f(0^-)=f(0)=f(0^+)$

$g'(0^+) = \lim_{x\to 0^+} f(x) + x f'(x) = f(0^+)$

$g'(0^-) = \lim_{x\to 0^-} f(x) + x f'(x) = f(0^-)$

$g'(0^-) = g'(0^+)$

$\Rightarrow$ g is differentiable at x = 0

(C) g is differentiable at x = 0, then f is continuous at x = 0

$g'(0^-) = g'(0^+)$

$\lim_{x\to 0} f(x) = \lim_{x\to 0^+} f(x)$

$f(0^-) = f(0^+)$ but not necessarily equal to f(0)

(D) g is discontinuous at x = 0 $\lim_{x\to 0} f(x)$ exists

Proved in option (C).

8. Consider the matrix $M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$. Let p, q, r, s, a, b, c and d be integers such that $M^{26} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $\sum_{k=1}^{26} M^k = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then which of the following statements is (are) TRUE?

(A) There exists a 2 × 2 invertible matrix N with real entries such that $MN = N\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(B) The value of a is 378

(C) For any two given integers m and n, then exist unique integers x and y such that px + qy = m and rx + sy = n

(D) For each positive real number t, the system of linear equations (a + t)x + by = 1 cx + (d + t)y = –1 has a unique solution

Ans (A), (C) and (D)

$M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

$M^{26} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $\sum_{k=1}^{26} M^k = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(A) $MN = N\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Let $N = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2a-c & 2b-d \\ a & b \end{bmatrix} = \begin{bmatrix} a & a+b \\ c & c+d \end{bmatrix}$

Comparing elements, $a = c, b = a + d, a = c, b = c + d$

$N = \begin{bmatrix} a & a+d \\ a & d \end{bmatrix} \Rightarrow |N| = ad - a^2 - ad - a^2 \neq 0$ possible if $a \neq 0$.

(B) $M = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow M^{26} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$$M^2 = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$$

$\vdots$

$$M^n = \begin{bmatrix} n+1 & -n \\ n & -(n-1) \end{bmatrix} M^{26} = \begin{bmatrix} 27 & -26 \\ 26 & -25 \end{bmatrix}$$

Comparing, $p = 27, q = -26, r = 26, s = -25$

$$\sum_{k=1}^{26} M^K = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a = \sum_{n=1}^{26}(n+1) = \frac{26 \cdot 27}{2} + 26 = 377$$

$$b = \sum_{n=1}^{26} -n = -351$$

$$c = \sum_{n=1}^{26} n = 351$$

$$d = -\sum_{n=1}^{26}(n-1) = -325$$

(C) The system of equations is $27x - 26y = m$ and $26x - 25y = n$

$$D = \begin{vmatrix} 27 & -26 \\ 26 & -25 \end{vmatrix} \neq 0 \Rightarrow \text{unique solution}$$

(D) The system of equations is

$(a+t)x + by = 1$

$cx + 1(d+t)y = -1$

$$D = \begin{vmatrix} a+t & b \\ c & d+t \end{vmatrix} = \begin{bmatrix} 377+t & -351 \\ 351 & -325+t \end{bmatrix}$$

$$= (377+t)(t-325) + 351 \times 351$$

$$= t^2 + 52t + 676 = (t+26)^2 > 0 \ \forall\ x \in R^+ \ \Rightarrow \text{unique solution.}$$

Section 3
Numerical Problems

9. Let S = {1, 2, 3, … , 10}. Consider the set X = {R : R is an equivalence relation on the set S such that R has exactly 42 elements}.

Then the number of elements in X is _____________.

Ans 2520

$S = \{1, 2, 3, 4, ..., 10\}$

X = {R : R is an equivalence relation on the set S such that R has exactly 42 elements}.

Let $n_1, n_2, n_3, \ldots$ be the number of elements in the partition.

$n_1 + n_2 + n_3 .... = 10$

$n_1^2 + n_2^2 + n_3^2 + ... = 42$

We need to think the non-negative integral solution of $n_1, n_2, n_3 ....$

First case is

$36+4+1+1=42$

$6^2+2^2+1^2+1^2=42$

$6+2+1+1=10$

$(6, 2, 1, 1)$

Total number of ways to divide 10 different object into 4 groups of group size 6, 2, 1, 1

$\frac{10!}{6!\,2!\,1!\,1!}\times\frac{1}{2!}$

Second case is

$25+16+1=42$

$5^2+4^2+1^2$

$5+4+1=10$

$\frac{10!}{5!\,4!\,1!}$

So, total number of ways will be

$=\frac{10!}{6!\,2!\,1!\,1!}\times\frac{1}{2!}+\frac{10!}{5!\,4!\,1!}$

$=\frac{10!}{5!}\left(\frac{1}{6\times 4}+\frac{1}{24}\right)$

$=10\times 9\times 8\times 7\times 6\times\frac{1}{12}$

$=2520$

10. Consider the function $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to(-\infty,\infty)$ defined by f(x) = (|x| + |x – 1|) sin x + [x sin x], where [x sin x] is the greatest integer less than or equal to x sin x.

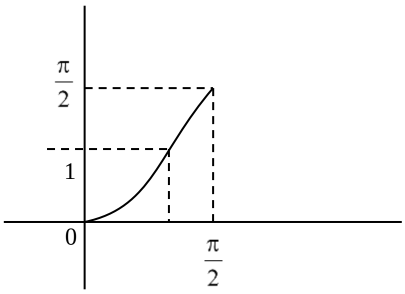
Let α be the total number of points in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ at which f is NOT continuous, and let β be the total number of points in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ at which f is NOT differentiable.

Then the value of α + β is _____________.

Ans 5

$f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to(-\infty,\infty)$

$f(x)=\underbrace{(|x|+|x-1|)\sin x}_{\text{Continuous}}+[x\sin x]$



Let $g(x)=[x\sin x]$

$h(x)=x\sin x$

$h(-x)=h(x)\ \Rightarrow$ h is an even function

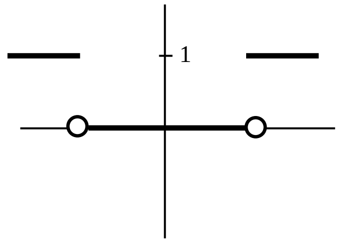
At x = 0, h(0) = 0

$h'(x)=x\cos x+\sin x$

$h'(x)>0$ in $\left(0,\frac{\pi}{2}\right)$

h is increasing in $\left(0, \frac{\pi}{2}\right)$

$h(0) = 0$ and $h\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$



So, the number of point of discontinuity will be 2

For point of non-differentiability

$f(x) = (|x| + |x-1|)\sin x + [x\sin x]$

$= |x|\sin x + |x-1|\sin x + [x\sin x]$

$x = 0$ and 1 are problematic points.

But at x = 0, sin 0 = 0 $\therefore$ differentiable and at x = 1, sin1 $\neq$ 0.

Hence, x = 1 is point of non-differentiability.

So, total number of points of non-differentiability is 3

11. The number of ways to distribute 10 identical red pens and 14 identical blue pens among four persons such that each person get 6 pens, is ____________.

Ans 206

Let the number of red pens given to the four people be r_1, r_2, r_3, r_4 and the number of blue pens be b_1, b_2, b_3, b_4.

$\sum_{i=1}^{4} r_i = 10$ and $\sum_{i=1}^{4} b_i = 14$

$r_i + b_i = 6 \Rightarrow b_i = 6 - r_i$

$b_i \geq 0 \Rightarrow 6 - r_i \geq 0 \Rightarrow r_i \leq 6$

$r_1 + r_2 + r_3 + r_4 = 10,\ 0 \leq r_i \leq 6$

Total number of ways without any restriction $= {}^{n+r-1}C_{r-1} = {}^{13}C_3 = 286$.

Total number of ways when one person gets atleast 7 red pens.

$r_1 + r_2 + r_3 + r_4 = 10,\ r_1 \geq 7$

First distribute 7 objects to 1st person.

Then we are left with 3 identical objects.

Which needs to be distributed among 4 persons without any restrictions.

$${}^{3+4-1}C_{4-1} = {}^{6}C_3 = \frac{6\times5\times4}{5\times6} = 20$$

Since, there are 4 people.

So, total number of unwanted ways = 4 × 20 = 80

Total number of valid ways = Total ways – unwanted ways = 286 – 80 = 206

12. Let $\alpha = \left(1-2\cos\left(\frac{\pi}{11}\right)\right)\left(1-2\cos\left(\frac{3\pi}{11}\right)\right)\left(1-2\cos\left(\frac{9\pi}{11}\right)\right)\left(1-2\cos\left(\frac{27\pi}{11}\right)\right)\left(1-2\cos\left(\frac{81\pi}{11}\right)\right)$.

Then the value of $5 - \alpha^2$ is ___________.

Ans 4

$$\alpha = \left(1-2\cos\left(\frac{\pi}{11}\right)\right)\left(1-2\cos\left(\frac{3\pi}{11}\right)\right)\left(1-2\cos\left(\frac{9\pi}{11}\right)\right)\left(1-2\cos\left(\frac{27\pi}{11}\right)\right)\left(1-2\cos\left(\frac{81\pi}{11}\right)\right)$$

Let $\theta = \frac{\pi}{11}$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta = \sin\theta(3-4\sin^2\theta)$$
$$= \sin\theta\left(3 - \frac{4(1-\cos 2\theta)}{2}\right) = \sin\theta(3-2+2\cos 2\theta)$$
$$= \sin\theta(1+2\cos 2\theta)$$
$$\frac{\sin 3\theta}{\sin\theta} = 1+2\cos 2\theta$$
$$\frac{\sin\left(\frac{3\theta}{2}\right)}{\sin\frac{\theta}{2}} = 1+2\cos\theta$$
$$\theta \to \pi-\theta \text{ gives } \frac{\sin\left(\frac{3\pi}{2}-\frac{3\theta}{2}\right)}{\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)} = \frac{-\cos\frac{3\theta}{2}}{\cos\frac{\theta}{2}} = 1-2\cos\theta$$
$$(1-2\cos\theta)(1-2\cos 3\theta)....(1-2\cos 81\theta) = \frac{\cos\frac{3\theta}{2}}{\cos\frac{\theta}{2}} \times \frac{\cos\frac{9\theta}{2}}{\cos\frac{3\theta}{2}} \times ..... \frac{\cos\frac{243\theta}{2}}{\cos\frac{81\theta}{2}}$$
$$= \frac{-\cos\left(\frac{243\theta}{2}\right)}{\cos\frac{\theta}{2}} - \frac{\cos\left(\frac{243\times\pi}{22}\right)}{\cos\left(\frac{\pi}{22}\right)} = \frac{-\cos\left(11\pi+\frac{\pi}{22}\right)}{\cos\left(\frac{\pi}{22}\right)} = 1$$

Section 4

Choose the appropriate entry/entries from List II to match each of the entries of the List I. It is possible that an option(s) in List II may be valid more than once, for a given entry in List I

13. Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		**List-II**
(P)	If α and β are the distinct roots of the equation $x^2 + x + 1 = 0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2026}}$ and $\frac{1}{(\beta+1)^{2026}}$ is	(1)	$x^2 + x + 1 = 0$
(Q)	If α and β are the distinct roots of the equation $x^2 + x + 1 = 0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2027}}$ and $\frac{1}{(\beta+1)^{2027}}$ is	(2)	$x^2 - x + 1 = 0$
(R)	If γ and δ are the distinct roots of the equation $x^2 - x + 1 = 0$, then the value of $\frac{1}{(\gamma-1)^{2026}} + \frac{1}{(\delta-1)^{2026}}$ is	(3)	$x^2 + x - 1 = 0$
(S)	If p and r are the distinct roots of the equation $x^2 + x - 1 = 0$, then the value of $\frac{1}{(p+1)^3} + \frac{1}{(r+1)^3}$ is	(4)	-1
		(5)	-4

(A) (P) → (1), (Q) → (2), (R) → (5), (S) → (4)
(B) (P) → (3), (Q) → (1), (R) → (4), (S) → (5)
(C) (P) → (1), (Q) → (2), (R) → (4), (S) → (5)
(D) (P) → (2), (Q) → (3), (R) → (5), (S) → (4)

Ans (C)

(P) $x^2+x+1=0$

$\alpha=\omega, \beta=\omega^2$

$$\frac{1}{(\alpha+1)^{2026}}=\frac{1}{\alpha^2}=\beta$$

$$\frac{1}{(\beta+1)^{2026}}=\frac{1}{(\beta^2)^{2026}}=\frac{1}{\beta^2}=\alpha$$

$\therefore$ equation is $x^2+x+1 \Rightarrow P\to 1$

(Q) $x^2+x+1=0$, α, β are roots

$$\frac{1}{(\alpha+1)^{2027}}=\frac{1}{(-\alpha^2)^{2027}}=-\frac{1}{\alpha}=-\alpha^2=\alpha+1$$

$$\frac{1}{(\beta+1)^{2027}}=\frac{1}{(-\beta^2)^{2027}}=\frac{-1}{\beta}=-\beta^2=\beta+1$$

Sum of roots $=(\alpha+1)+(\beta+1)=\alpha+\beta+2=1$

Product of roots $=(\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1=1$

$\therefore$ Equation is $x^2-x+1=0 \quad \Rightarrow Q\to 2$.

(R) $x^2-x+1=0$

$\gamma=-\omega, \delta=-\omega^2$

$$\frac{1}{(\gamma-1)^{2026}}+\frac{1}{(\delta-1)^{2026}}=\frac{1}{(-\omega-1)^{2026}}+\frac{1}{(-\omega^2-1)^{2026}}$$

$$=\frac{1}{(\omega^2)^{2026}}+\frac{1}{(\omega)^{2026}}=\frac{1}{\omega^2}+\frac{1}{\omega}$$

$$=\omega+\omega^2=-1\Rightarrow R\to 4$$

(S) $x^2+x-1=0\Rightarrow p+r=-1, pr=-1$

$$\frac{1}{(p+1)^3}+\frac{1}{(r+1)^3}=\frac{1}{(-r)^3}+\frac{1}{(-p)^3}$$

$$=p^3+r^3=(p+r)^3-3pr(p+r)$$

$$=-1-3(-1)(-1)=-4 \Rightarrow S\to 5$$

14. Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I		List-II	
(P)	The number of elements in the set $\{x \in [-\pi, \pi] : \sin^6 x + \cos^4 x = 1\}$	(1)	is 1
(Q)	The number of elements in the set $\left\{x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]: \sin^2 x+\cos^6 x=1\right\}$	(2)	is 2
(R)	The number of elements in the set $\left\{x\in[-\pi,\pi]:\cos^2\left(\frac{x}{2}\right)-\sin^2 x=\frac{1}{2}\right\}$	(3)	is 3

(S)	The number of elements in the set $\left\{x \in [-2\pi, 2\pi] : 6\sin^2\left(\frac{x}{2}\right) - \cos 3x = 3\right\}$	(4)	is 4
		(5)	is 5

(A) (P) → (2), (Q) → (5), (R) → (3), (S) → (4)
(B) (P) → (5), (Q) → (3), (R) → (2), (S) → (4)
(C) (P) → (5), (Q) → (4), (R) → (1), (S) → (3)
(D) (P) → (4), (Q) → (3), (R) → (2), (S) → (5)

Ans (B)

(P) $x \in [-\pi, \pi]$

$\sin^6 x + \cos^4 x = 1 \Rightarrow \sin^6 x + (1 - \sin^2 x)^2 = 1$

$\Rightarrow \sin^6 x + \sin^4 x - 2\sin^2 x = 0$

$\Rightarrow \sin^2 x(\sin^4 x + \sin^2 x - 2) = 0$

$\Rightarrow \sin^2 x(\sin^2 x + 2)(\sin^2 x - 1) = 0$

$\Rightarrow \sin^2 x = 0,\ \sin^2 x = -2 \text{ (invalid)},\ \sin^2 x = 1$

$\Rightarrow x = 0, \pm\pi \qquad x \pm \frac{\pi}{2}$

as $x \in [-\pi, \pi] \rightarrow$ 5solutions $\Rightarrow P \rightarrow 5$

(Q) $\sin^2 x + \cos^6 x = 1,\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \cos^2 x(\cos^2 x + 2)(\cos^2 x - 1) = 0$

$\Rightarrow \cos^2 x = 0,\ \cos^2 x = 1$

$\Rightarrow x = \pm\frac{\pi}{2},\ x = 0$

3 solution $\Rightarrow Q \rightarrow 3$

(R) $x \in [-\pi, \pi]$

$\cos^2\frac{x}{2} - \sin^2 x = \frac{1}{2}$

$\Rightarrow \frac{1}{2}(1 + \cos x) - (1 - \cos^2 x) = \frac{1}{2}$

$\Rightarrow 1 + \cos x - 2 + 2\cos^2 x = 1$

$\Rightarrow 2\cos^2 + \cos x - 2 = 0$

$\Rightarrow \cos^2 x + \frac{1}{2}\cos x - 1 = 0$

$\Rightarrow \left(\cos x + \frac{1}{4}\right)^2 - \frac{17}{16} = 0$

$\Rightarrow \left(\cos x + \frac{1}{4}\right)^2 = \frac{17}{16}$

$\Rightarrow \cos x = \frac{-1}{4} \pm \frac{\sqrt{17}}{4}$

As $\frac{-1-\sqrt{17}}{4} < -1 \Rightarrow \cos x = \frac{-1-\sqrt{17}}{4}$ is rejected and $0 < \frac{-1+\sqrt{17}}{4} < 1 \Rightarrow \cos x = \frac{-1+\sqrt{17}}{4}$ is accepted. $\cos(-x) = \cos x$. only two solutions $\Rightarrow R \rightarrow (2)$

(S) $x \in [-2\pi, 2\pi]$

$$6\sin^2\frac{x}{2}-\cos 3x=3$$
$$3(1-\cos x)-(4\cos^3 x-3\cos x)-3=0$$
$$\Rightarrow 4\cos^3 x=0 \Rightarrow \cos x=0$$
$$x=\pm\frac{\pi}{2},\pm\frac{3\pi}{2}\Rightarrow S\to 4$$
$\therefore$ B is correct.

15. For real numbers α, β, γ, δ and μ, consider the matrix $M=\begin{bmatrix}\alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}}\\ \gamma & \delta & \mu\end{bmatrix}$. Suppose that $MM^T = I$, where M^T is the transpose of the matrix M, and I is the 3 × 3 identity matrix. Let $\vec{u}=\alpha\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\gamma\hat{k}$, $\vec{v}=\frac{1}{\sqrt{2}}\hat{i}+\beta\hat{j}+\delta\hat{k}$ and $\vec{w}=-\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\mu\hat{k}$.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		**List-II**
(P)	The value of $\gamma^2+\delta^2$ is	(1)	0
(Q)	If $x\vec{u}+y\vec{v}+z\vec{w}=\hat{j}$ for some real numbers x, y and z, then the value of x is	(2)	1
(R)	The value of $\left|\vec{u}\cdot(\vec{v}\times\vec{w})\right|$ is	(3)	$\frac{1}{\sqrt{2}}$
(S)	The value of $\left|\vec{u}\times(\vec{v}\times\vec{w})\right|$ is	(4)	$\frac{1}{\sqrt{3}}$
		(5)	$\frac{5}{6}$

(A) (P) → (5), (Q) → (4), (R) → (2), (S) → (1)
(B) (P) → (4), (Q) → (5), (R) → (1), (S) → (2)
(C) (P) → (5), (Q) → (3), (R) → (2), (S) → (1)
(D) (P) → (5), (Q) → (4), (R) → (1), (S) → (2)

Ans (A)

$$M=\begin{bmatrix}\alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}}\\ \gamma & \delta & \mu\end{bmatrix}$$
$$\vec{u}=\alpha\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\delta\hat{k}$$
$$\vec{v}=\frac{1}{\sqrt{2}}\hat{i}+\beta\hat{j}+\delta\hat{k}$$
$$\vec{w}=\frac{-1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\mu\hat{k}$$
$M\cdot M^T=I\Rightarrow$ M is orthogonal

$\Rightarrow |\vec{u}| = |\vec{v}| = |\vec{w}| = 1$

$\Rightarrow \vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{w} = \vec{\omega}\cdot\vec{u} = 0$

$\Rightarrow \alpha^2 + \frac{1}{3} + \delta^2 = 1, \frac{1}{2} + \beta^2 + \delta^2 = 1, \frac{1}{2} + \frac{1}{3} + \mu^2 = 1$

$\Rightarrow \alpha^2 + \delta^2 = \frac{2}{3}, \beta^2 + \gamma^2 = \frac{1}{2}, \mu^2 = \frac{1}{6}$

(P) Compare a_{33} element of $MM^T = I$

$\gamma^2 + \delta^2 + \mu^2 = 1 \Rightarrow \gamma^2 + \delta^2 = 1 - \mu^2 = \frac{5}{6} \quad \Rightarrow P \to 5$

(Q) $x\vec{u} + \gamma\vec{v} + z\vec{w} = \hat{j}$

dot with $\vec{u} \Rightarrow x(\vec{u}\cdot\vec{u}) = \vec{u}\cdot\hat{j}$

$\Rightarrow x = \frac{1}{\sqrt{3}} \Rightarrow (Q) \to (4)$

(R) $|\vec{u}\cdot(\vec{v}\times\vec{w})| = 1$ are $\vec{u}, \vec{v}, \vec{w}$ are orthogonal vectors $\Rightarrow R \to (2)$

(S) $|\vec{u}\times(\vec{v}\times\vec{w})|$

$\vec{u}\times(\vec{v}\times\vec{w}) = (\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w}$

$= \vec{0}$ as $\vec{u}\cdot\vec{w} = \vec{v}\cdot\vec{u} = \vec{0}$

$\therefore S \to 1 \qquad \therefore$ A is correct.

16. Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		List-II
(P)	The circle with centre (1, 2) and touching the straight line 3x + 4y = 1, passes through	(1)	the point (1, 1)
(Q)	The common tangent to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ with positive slope, passes through	(2)	the point (7, 9)
(R)	Let M be the end point of the latus rectum of the ellipse $3x^2 + 4y^2 = 48$ such that M lies in the first quadrant. Then the normal to the ellipse drawn at M passes through	(3)	the point (3, 2)
(S)	Let H be the hyperbola whose centre is at the origin, one of the foci is at (5, 0) and one directrix is 5x + 16 = 0. Then H passes through	(4)	the point (2, 5)
		(5)	the point $(8, 3\sqrt{3})$

(A) (P) → (3), (Q) → (4), (R) → (1), (S) → (2)
(B) (P) → (3), (Q) → (2), (R) → (1), (S) → (5)
(C) (P) → (3), (Q) → (2), (R) → (4), (S) → (5)
(D) (P) → (4), (Q) → (1), (R) → (2), (S) → (3)

Ans (B)

(P) $r = \left|\frac{3+8-1}{5}\right| = 2$

Equation of circle is $(x-1)^2 + (y-2)^2 = 4$ passes through $(3, 2) \Rightarrow P \to 3$.

(Q) $x^2 + y^2 = 2, y^2 = 8x$

Tangent to $y^2 = 8x$ is $y = mx + \frac{2}{m}$

This is also tangent to circle $\Rightarrow \left(\frac{2}{m}\right)^2 = 2(1+m^2)$

$\Rightarrow \dfrac{2}{m^2} = 1 + m^2 \Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow (m^2 - 1)(m^2 + 2) = 0 \Rightarrow m = \pm 1$

$m > 0 \Rightarrow m = 1 \;\; \therefore y = x + 2$ is the tangent

Passes through (7, 9) $\Rightarrow Q \to 2$

(R) $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$

$M\left(2, \dfrac{12}{4}\right) = (2, 3)$

Equation of normal at $(2, 3)$ is $\dfrac{16x}{2} - \dfrac{12y}{3} = 16 - 12$

$\Rightarrow 8x - 4y = 4 \Rightarrow 2x - y = 1$; (1, 1) is satisfied $\Rightarrow R \to 1$.

(S) $ae = 5, \dfrac{a}{e} = \dfrac{16}{5} \Rightarrow a^2 = 16, b^2 = a^2e^2 - a^2 = 25 - 16 = 9$

$\therefore \dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$; $(8, 3\sqrt{3})$ is satisfied. $S \to 5$

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